C2 Exercise 1C (algebraic division with remainders)

Note Title 16/01/2007

1a divide $x^3 + x + 10$ by $(x+2)$

$$
\begin{array}{r}
x^2 - 2x + 5 \\
\hline
x+2 \;\big)\; x^3 + 0x^2 + x + 10 \\
-[x^3 + 2x^2] \quad\downarrow \\
\hline
-2x^2 + x \\
-[-2x^2 - 4x] \\
\hline
5x + 10 \\
5x + 10 \\
\hline
0
\end{array}
$$

CZ Ex 1C

16 $\dfrac{2x^3 - 17x + 3}{x+3}$

$$
\begin{array}{r}
2x^2 - 6x + 1 \\
x+3 \overline{)\,2x^3 + 0x^2 - 17x + 3} \\
-[2x^3 + 6x^2] \\
\hline
-6x^2 - 17x \\
-[-6x^2 - 18x] \\
\hline
+x + 3 \\
-[+x + 3] \\
\hline
0
\end{array}
$$

Ex C2

1c $\dfrac{-3x^3 + 50x - 8}{x - 4}$

$$
\begin{array}{r}
-3x^2 - 12x + 2 \\
x - 4 \overline{)\,-3x^3 + 0x^2 + 50x - 8} \\
\underline{-(-3x^3 + 12x^2)} \\
-12x^2 + 50x \\
\underline{-(-12x^2 + 48x)} \\
2x - 8 \\
\underline{-(2x - 8)} \\
0
\end{array}
$$

C2 Ex 1C

2a $\dfrac{x^3 + x^2 - 36}{x - 3}$

$$
\begin{array}{r}
x^2 + 4x + 12 \\
x-3 \;\overline{)\; x^3 + x^2 + 0x - 36} \\
-[x^3 - 3x^2] \\
\hline
4x^2 + 0x \\
-[4x^2 - 12x] \\
\hline
12x - 36 \\
-[12x - 36] \\
\hline
0
\end{array}
$$

C2 Ex 1C

2b $\dfrac{2x^3 + 9x^2 + 25}{x+5}$

$$
\begin{array}{r}
2x^2 - x + 5 \\
x+5 \;\overline{)\; 2x^3 + 9x^2 + 0x + 25} \\
\underline{-[2x^3 + 10x^2]} \\
-x^2 + 0x \\
\underline{-[-x^2 - 5x]} \\
5x + 25 \\
\underline{-[5x + 25]} \\
0
\end{array}
$$

C2 Ex 1C

2c $\dfrac{-3x^3 + 11x^2 - 20}{(x-2)}$

$$
\begin{array}{r}
-3x^2 + 5x + 10 \\
x-2 \;\overline{)\; -3x^3 + 11x^2 + 0x - 20} \\
-[-3x^3 + 6x^2] \\
\hline
5x^2 + 0x \\
-[5x^2 - 10x] \\
\hline
10x - 20 \\
-[10x - 20] \\
\hline
0
\end{array}
$$

C2 Ex 1C

3a $\dfrac{x^3+2x^2-5x-10}{x+2} = \dfrac{f(x)}{x+2}$. find $f(-2) = -8+8+10-10 = 0$ so there's no remainder

$$
\begin{array}{r}
x^2+0x-5 \\
x+2\,\overline{)\,x^3+2x^2-5x-10} \\
-[x^3+2x^2] \\
\hline
0x^2-5x \\
-[0x^2+0x] \\
\hline
-5x-10
\end{array}
$$

So $\dfrac{x^3+2x^2-5x-10}{x+2} = x^2-5$

C2 Ex 1C

3b $\dfrac{2x^3 - 6x^2 + 7x - 21}{x-3}$

$f(3) = 54 - 54 + 21 - 21 = 0$

$$
\begin{array}{r}
2x^2 + 0x + 7 \\
x-3 \overline{)\,2x^3 - 6x^2 + 7x - 21} \\
\underline{-\,[2x^3 - 6x^2]} \\
0x^2 + 7x \\
\underline{-\,[0x^2 + 0x]} \\
7x - 21 \\
\underline{-\,[7x - 21]} \\
0
\end{array}
$$

$$\frac{2x^3 - 6x^2 + 7x - 21}{x-3} = 2x^2 + 7$$

C2 Ex 1C

3c $\dfrac{-3x^3 + 21x^2 - 4x + 28}{x - 7}$

$$
\begin{array}{r}
-3x^2 + 0x - 4 \\
\hline
x - 7 \;\big)\; -3x^3 + 21x^2 - 4x + 28 \\
\underline{-(-3x^3 + 21x^2)} \\
0x^2 - 4x \\
\underline{0x^2 + 0x} \\
-4x + 28 \\
\underline{-(-4x + 28)} \\
0
\end{array}
$$

$$\frac{-3x^3 + 21x^2 - 4x + 28}{x - 7} = -3x^2 - 4$$

C2 Ex 1C

4 Find the remainder when:

a

$$\frac{x^3 + 4x^2 - 3x + 2}{x+5} = \frac{f(x)}{x+5}$$

. find $f(-5) = -125 + 100 + 15 + 2$
$= -8$

$$x+5 \overline{)\, x^3 + 4x^2 - 3x + 2}$$

$x^2 - x + 2$

$x^3 + 5x^2$

$-x^2 - 3x$

$-[-x^2 - 5x]$

$2x + 2$

$-[2x + 10]$

-8

So remainder is -8

ie $\frac{f(x)}{x+5} = x^2 - x + 2 - \frac{8}{x+5}$

CZ Ex 1C

46 find the remainder when you do $(3x^3 - 20x^2 + 10x + 5) \div (x - 6)$

$f(6) = 3 \times 216 - 20 \times 36 + 10 \times 6 + 5$

$= 648 - 720 + 60 + 5 = 713 - 720 = -7$

$$
\begin{array}{r}
3x^2 - 2x - 2 \\
x - 6 \overline{)\,3x^3 - 20x^2 + 10x + 5} \\
-[3x^3 - 18x^2] \\
\hline
-2x^2 + 10x \\
-[-2x^2 + 12x] \\
\hline
-2x + 5 \\
-2x + 12 \\
\hline
-7
\end{array}
$$

remainder

$$\frac{3x^3 - 20x^2 + 10x + 5}{x - 6} = 3x^2 - 2x - 2 + \frac{-7}{x - 6}$$

C2 Ex 1C

4c find the remainder when $-2x^3 + 3x^2 + 12x + 20$ is divided by $(x-4)$

Substitute $x = 4 \Rightarrow f(x) = -2(64) + 3(16) + 12(4) + 20$

$= -128 + 48 + 48 + 20 = -12$

$$
\begin{array}{r}
-2x^2 - 5x - 8 \\
x-4 \;\overline{)\; -2x^3 + 3x^2 + 12x + 20} \\
-[-2x^3 + 8x^2] \\
\hline
-5x^2 + 12x \\
-[-5x^2 + 20x] \\
\hline
-8x + 20 \\
-8x + 32 \\
\hline
-12
\end{array}
$$

So the remainder is -12 when you divide $f(x)$ by $(x-4)$

C2 Ex1C

5 Show that when $3x^3 - 2x^2 + 4$ is divided by $(x-1)$ the remainder is 5

$$
\begin{array}{r}
3x^2 + x + 1 \\
x-1 \;\overline{)\; 3x^3 - 2x^2 + 4} \\
-\left[3x^3 - 3x^2\right] \\
\hline
x^2 + 0x \\
-\left[x^2 - x\right] \\
\hline
x + 4 \\
-\left[x - 1\right] \\
\hline
5
\end{array}
$$

C2 Ex 1C

6 Show that when $3x^4 - 8x^3 + 10x^2 - 3x - 25$ is divided by $(x+1)$ the remainder is -1

$$
\begin{array}{r}
3x^3 - 11x^2 + 21x - 24 \\
\hline
x+1 \;\big)\; 3x^4 - 8x^3 + 10x^2 - 3x - 25 \\
-\left[3x^4 + 3x^3\right] \\
\hline
-11x^3 + 10x^2 \\
-\left[-11x^3 - 11x^2\right] \\
\hline
21x^2 - 3x \\
-\left[21x^2 + 21x\right] \\
\hline
-24x - 25 \\
-\left[-24x - 24\right] \\
\hline
-1
\end{array}
$$

C2 Ex 1C

7 Show that $(x+4)$ is a factor of $5x^3 - 73x + 28$

$$
\begin{array}{r}
5x^2 - 20x + 7 \\
x+4 \;\overline{)\; 5x^3 + 0x^2 - 73x + 28} \\
-[5x^3 + 20x^2] \\
\hline
-20x^2 - 73x \\
-[-20x^2 - 80x] \\
\hline
7x + 28 \\
7x + 28 \\
\hline
0
\end{array}
$$

Since no remainder $(x+4)$ is a factor

C2 Ex 1C

8 Simplify $\dfrac{3x^3 - 8x - 8}{x - 2}$

$$
\begin{array}{r}
3x^2 + 6x + 4 \\
x-2 \overline{)\,3x^3 + 0x^2 - 8x - 8} \\
-[3x^3 - 6x^2] \\
\hline
6x^2 - 8x \\
-[6x^2 - 12x] \\
\hline
4x - 8 \\
-[4x - 8] \\
\hline
0
\end{array}
$$

So $\dfrac{3x^3 - 8x - 8}{x - 2} \equiv 3x^2 + 6x + 4$

C2 Ex 1C

9 Divide $x^3 - 1$ by $(x-1)$

$(x-1)(x^2+x+1)$ by inspection so $\frac{x^3-1}{x-1} = x^2+x+1$

$$
\begin{array}{r}
x^2 + x + 1 \\
x-1 \;\overline{)\; x^3 + 0x^2 + 0x - 1} \\
\underline{-[x^3 - x^2]} \\
x^2 + 0x \\
\underline{-[x^2 - x]} \\
x - 1 \\
\underline{-[x - 1]} \\
0
\end{array}
$$

C2 Ex 1C

10 Divide $x^4 - 16$ by $(x+2)$

$(x+2)(x^3 - 2x^2 + 4x - 8)$ by inspection [I've looked at it and factorised it in my head]...

$$
\begin{array}{r}
x^3 - 2x^2 + 4x - 8 \\
x+2 \overline{)\, x^4 - 0x^3 + 0x^2 + 0x - 16} \\
-[x^4 + 2x^3] \\
\hline
-2x^3 - 0x^2 \\
-[-2x^3 - 4x^2] \\
\hline
4x^2 + 0x \\
-[4x + 8x] \\
\hline
-8x - 16 \\
-[-8x - 16] \\
\hline
0
\end{array}
$$